What has the manager done for me? A value-based solution to the measurement of fund performance in relation to a benchmark

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Abstract

This paper presents a method of performance measurement and attribution analysis which is based on values rather than returns. The difference in final values between a managed fund and its benchmark provides an exact measure of the contribution of the fund manager to the wealth of investors. It is natural for investors to be interested in the manager's contribution, as well as in the manager's skill. The paper shows that working with values is relatively simple and transparent, that it is free from most of the problems of multiperiod attribution analysis, and that it enables precise customized analyses to be made of the manager's contribution for each investor in a fund.

Introduction: the case for values

Attribution analysis of portfolio performance was initially developed in the setting of a single time period, but the important question of how to conduct the analysis if there is more than one period has attracted much attention in recent years. In most of the work on single-period and multiperiod attribution, and in current industry practice, it is taken for granted that outcomes are measured by percentage returns. The current paper presents an alternative approach to performance measurement and attribution analysis which is based on values rather than returns. We shall argue that there are two merits to a value-based approach. First, values provide an exact measure of the contribution of the fund manager to the wealth of an investor, whereas in general the manager's contribution can not be measured exactly using returns. Second, multiperiod attribution analysis is easier using values than it is using returns.

If there is a single assessment period (with no cash flows during the period), performance measurement and attribution analysis can equally well be conducted using returns or values, because the two are arithmetically equivalent. The challenge if there are $T > 1$ periods has been to account for the difference between the compound or 'time-weighted' return (TWR) on a portfolio P, R_P, and the compound return on P's benchmark B, R_B, where

$$R = \Pi_{t=1}^{T}(1 + R_t) - 1,$$

and R_t is the percentage return in period t.

A key point about the TWR is that it is unaffected by the amounts of money an investor injects or withdraws at various dates. The return for each period is assigned an equal weight in the calculation of the TWR. The main argument for using TWRs rests on the assumptions that the purpose of the analysis is to assess the manager's skill, and that the manager does not influence the investor's decisions regarding cash inflows and outflows (for example, Sharpe, Alexander and Bailey, 1999, pp. 827-8). Given these assumptions, it is appropriate to assign the portfolio's return for each period an equal weight. It is also helpful to be able to compare the TWRs across funds which have had different patterns of cash inflows and outflows.

However, a portfolio's TWR will not explain the difference between the value of the fund at the start date 0, V_{P0}, and the value at the end date T, V_{PT}, if investors make cash injections or withdrawals during the T periods. In other words, $V_{PT} \neq V_{P0}(1 + R)$ if there are intervening cash flows (and it is perfectly possible for the TWR on a fund to have the opposite sign to the value added or lost by the manager). As a result, the performance in relation to a benchmark measured by TWRs, $R_P - R_B$, is not equivalent to the performance measured by values, $V_{PT} - V_{BT}$. A related point is that, if there is more than one investor in a managed fund, there will be separate measures of performance in terms of values for each investor's holding and for the whole fund. In contrast, performance measured by TWRs has to be the same for each investor as it is for the fund, whatever the differences across investors in the cash injections and withdrawals they have made.

We believe that there is a strong case for measuring performance by values as well as by TWRs, because we believe that investors are interested in the manager's contribution to the value of their investment, as well as in the verdict on the manager's skill. A natural question for an investor to ask is 'What has the manager done for me, given my initial investment and the cash inflows and outflows by me along the way?' This is asking for something different from a judgement regarding the manager's skill. The difference in final values, $V_{PT} - V_{BT}$, is an exact measure of the manager's contribution to the value of the fund, whereas the difference in TWRs, $R_P - R_B$, is not an exact measure, as we have just observed. Likewise, attribution analysis of the value added or lost is of interest to investors, as it provides a better understanding of how the gain or loss in value has arisen. The methods offered to date for multiperiod attribution are seeking to explain $R_P - R_B$ or $(1 + R_P)/(1 + R_B)$, not $V_{PT} - V_{BT}$. Hence none of them can explain correctly the contribution of the manager to portfolio value, if there are intervening cash flows. Attribution analysis of the manager's contribution requires a different methodology.

A second point about TWRs is that attribution analysis is awkward: hence the flurry of recent papers on the subject. The problem is that conventional attribution analysis, developed to explain a single-period difference in returns, $R_{Pt} - R_{Bt}$, can not be applied without modification to explain $R_P - R_B$ (whether or not there are intervening cash flows by investors). Two approaches have been developed to deal with this. The first approach is based on 'arithmetic linking', ie adding the difference in returns for each period. The differences do not add up to equal the difference in the TWRs:

$$R_P - R_B \neq \Sigma_{t=1}^{T} (R_{Pt} - R_{Bt}).$$

So various methods of adjusting the returns for each period have been proposed, as a result of which the sum of the differences is made equal to $R_P - R_B$. But the adjustments introduce distortions into the results of attribution analysis, and reduce its transparency.[1] The second approach is to use 'geometric linking'. Performance in a given period t is measured by

$$G_t = (1 + R_{Pt})/(1 + R_{Bt}) - 1,$$

rather than by $R_{Pt} - R_{Bt}$. The attraction of geometric linking is that there is an exact multiplicative relation between the unadjusted performance measure for each period and the measure for all T periods:

$$(1 + R_P)/(1 + R_B) = \Pi_{t=1}^{T}(1 + G_t).$$

Because of this feature, an exact attribution analysis for $(1 + R_P)/(1 + R_B)$ can easily be carried out at the level of the fund, ie in terms of the total effects of each of asset allocation, stock selection and the interaction between the two. It is harder to measure the multiperiod effects of the attributes for a particular asset class.[2] The awkwardness of attribution analysis of TWRs is such that consensus has yet to be reached regarding the best method.

We show in the paper that multiperiod attribution analysis in terms of values can be done precisely for the whole fund and for each asset class, whether or not there are intervening cash flows, without making any adjustments to the end-date values or to the returns in any period. The analysis is therefore relatively simple and transparent, in comparison with the analysis in terms of TWRs. The value-based method is also very flexible. Attribution can be taken down to the level of individual assets, if desired. The contribution can be calculated from not holding assets which are in the benchmark, and from holding assets which are not in the benchmark. The method can accommodate short positions and cash flows associated with expenses and taxes. It gives correct answers for *any* type of fund, including multicurrency funds, hedge funds and funds using derivatives - all of which are notorious for causing problems for performance measurers. We provide a worked example of one of these problem cases, a multicurrency fund.

It should be noted that the value-based method involves money weighting, in that the impact of the manager's investment decisions each period on the final value of the fund is positively related to the value of the fund at the start of the period. But the method does not use money-weighted returns as commonly understood,[3] nor does it use internal rates of return.[4] We shall first explain the method in general terms, and then present two examples.

The value-based method

Measurement of value added or lost

We assume that performance is to be assessed over a fixed length of time in which there is more than one period. It is, of course, very important to agree in advance the dates between which the manager will be assessed, since the length of time is likely to affect the manager's behavior. The manager's contribution to the value of the actively managed portfolio P is measured by comparing the end-date market value of the portfolio, V_{PT}, with the notional end-date value of P's benchmark portfolio, V_{BT}. The difference can be expressed, if desired, as a percentage of the benchmark's final value: $(V_{PT} - V_{BT})/V_{BT}$. This relative performance measure indicates how much value the manager has gained or lost per final dollar which would have arisen from a policy of investing in the benchmark.

The final value of portfolio P can be observed directly. The final value arising from a policy of investing in the benchmark B needs to be calculated, and this can be done as follows. A notional investment is made in B of the same amount as was invested in P at date 0, so that $V_{P0} = V_{B0}$. At any given date, including date 0, P may hold assets or asset classes in proportions which differ from those in B, and the constituents may differ; P may have zero holdings of some assets in B, and positive holdings of other assets not in B. We shall assume that the benchmark's composition is given by the weights of its holdings in each of N asset classes, but the composition could alternatively be given by the weights for individual assets. Each date on which there is a cash inflow or outflow for P, a matching notional cash flow is made for B. Purchases or sales in each asset class are made in line with their weights in the benchmark at date t. So the cash inflow or outflow at date t for asset class A in the benchmark is given by $Y_{BAt} = w_{BAt}Y_t$, where Y_t is a cash flow for P and B at date t (negative in the case of a cash outflow) and w_{BAt} is the weight of A in the benchmark. Y_0 is the amount of the initial investment. The notional return on the benchmark in period t is given by

$$\begin{aligned} R_{Bt} &= \Sigma_{A=1}^{N} w_{BAt-1}(V_{BAt}/V_{BAt-1} - 1) \\ &= \Sigma_{A=1}^{N} w_{BAt-1}R_{BAt}, \end{aligned}$$

where V_{BAt} is the benchmark's holding in asset class A at date t, including any cash income received.[5] The notional final value of the benchmark is calculated from

$$V_{BT} = \Sigma_{t=0}^{T} Y_t[1 + R_B(t, T)],$$

where $R_B(t, T)$ is the compound return on the benchmark between dates t and T:

$$R_B(t, T) = \Pi_{\tau=t}^{T}(1 + R_{B\tau}) - 1.$$

V_{BT} is the portfolio's notional final value if the same initial investment and the same subsequent inflows and outflows were made in the benchmark as were made in the portfolio. V_{BT} therefore incorporates the effects of the timing and size of the cash flows, were a passive investment policy being followed. This is appropriate because such cash flows are the responsibility of the investor, not the manager. Any difference between the final values of the portfolio and its benchmark is due to the actions of the manager.

The composition of the benchmark is not affected by the timing or amounts of cash flows by investors. But the composition will change over time due to changes in asset prices and to re-investment of income. An alternative conception of the benchmark is that the weights are *fixed* over time. This implies that the benchmark is rebalanced at each date to ensure that weights remain constant, and that cash inflows or outflows are made by buying or selling asset classes in proportion to their constant weights. The question of whether to assume that the benchmark is rebalanced arises in the multiperiod context whether one uses TWRs or our proposed value-based method. An assumption of rebalancing raises the problem of the associated extra transactions costs. We have assumed that the weights of asset classes in the benchmark are allowed to drift over time from their weights at date 0.

With no injections or withdrawals of cash by investors, the relative performance measure based on values is equivalent to the measure based on TWRs:

$$(V_{PT} - V_{BT})/V_{BT} \;=\; (R_P - R_B)/(1 + R_B),$$

since $V_T = V_0(1 + R)$ and $V_{P0} = V_{B0}$. The value-based measure comes into its own if investors make injections or withdrawals of cash. In this case, the relative performance measure for the portfolio based on values, $(V_{PT} - V_{BT})/V_{BT}$, is not the same conceptually or numerically as the measure based on TWRs, $(R_P - R_B)/(1 + R_B)$. In addition, each investor's holding will have a distinct relative performance measure using values. The value added or lost by the manager for a given investor i in P is measured in a way analogous to the way the value added for P is measured. The manager's contribution for i is measured by $V_{PT,i} - V_{BT,i}$, where $V_{PT,i}$ is the final market value of i's holding in P and $V_{BT,i}$ is the final value of i's notional holding in the benchmark. Each date t on which there is a cash inflow or outflow by i, a matching notional cash flow is made for B and for i's holding in B. $V_{BT,i}$ is calculated from

$$V_{BT,i} \;=\; \Sigma_{t=0}^{T}\, Y_{t,i}[1 + R_B(t, T)],$$

where $Y_{t,i}$ is the cash inflow or outflow by i at date t. i's holding in B will always have the same composition as B and will earn the same return as B each period. But differences in the size and timing of cash flows across investors will mean that, in general, the manager's contribution per dollar in the benchmark will be different for any two investors *i* and *j* and for the whole portfolio:

$$(V_{PT} - V_{BT})/V_{BT} \neq (V_{PT,i} - V_{BT,i})/V_{BT,i} \neq (V_{PT,j} - V_{BT,j})/V_{BT,j}.$$

This is ignored in performance measurement using TWRs, which relates to the whole portfolio, not to the holdings of specific investors.

Attribution analysis

The manager's contribution to value can be analyzed in terms of asset allocation policy, stock selection and an interaction term, in a manner analogous to the analysis of Brinson et al (1986, 1991). The formulas for asset class *A* for a single period given in Brinson et al (1991, p. 47) are reproduced here for convenience:

$$\begin{aligned} \text{asset allocation} &= (w_{PA} - w_{BA})(R_{BA} - R_B) \\ \text{stock selection} &= w_{BA}(R_{PA} - R_{BA}) \\ \text{interaction} &= (w_{PA} - w_{BA})(R_{PA} - R_{BA}) \\ \text{sum of attributes} &= w_{PA}R_{PA} - w_{BA}R_{BA} - (w_{PA} - w_{BA})R_B. \end{aligned} \tag{1}$$

The sum of the attributes is the contribution to $R_P - R_B$ relating to the holding in *A*, expressed as a return. The same contribution expressed as an increment to value, $V_{added}(V_{PA})$, is

$$V_{added}(V_{PA}) = V_{PA}R_{PA} - V_{BA}R_{BA} - (V_{PA} - V_{BA})R_B.$$

The contribution is the change in the difference between the holdings of *P* and *B* in *A*, less the change which would have arisen had the return on *A* in *P* been the same as the overall return on the benchmark.

Suppose now that there are two periods. The final value added or lost relating to the initial cash in *A*, $V_{added}(Y_{PA0})$, is given by

$$V_{added}(Y_{PA0}) = Y_{PA0}R_{PA}(0, 2) - Y_{BA0}R_{BA}(0, 2) - (Y_{PA0} - Y_{BA0})R_B(0, 2).$$

The final value added or lost relating to any cash flow for *A* at date 1 is given by

$$V_{added}(Y_{PA1}) = Y_{PA1}R_{PA}(1, 2) - Y_{BA1}R_{BA}(1, 2) - (Y_{PA1} - Y_{BA1})R_B(1, 2).$$

It can be seen that, with *T* periods, the total value added or lost relating to the cash flows for *A* is given by

$$\Sigma_{t=0}^{T} V_{added}(Y_{PAt}) = \Sigma_{t=0}^{T} [Y_{PAt}R_{PA}(t, T) - Y_{BAt}R_{BA}(t, T) - (Y_{PA1} - Y_{BA1})R_B(t, T)].$$

The final value added relating to the cash for A at each date can be analyzed via formulas which match those in equations (1). The sum of the values for each attribute across the dates gives the total value added for each attribute for A:

$$\begin{aligned} \text{asset allocation} \;=\; AA_{\mathrm{A}} &= \Sigma_{\mathrm{t=0}}{}^{\mathrm{T}}\,(Y_{\mathrm{PAt}} - Y_{\mathrm{BAt}})[R_{\mathrm{BA}}(t, T) - R_{\mathrm{B}}(t, T)] \\ \text{stock selection} \;=\; SS_{\mathrm{A}} &= \Sigma_{\mathrm{t=0}}{}^{\mathrm{T}}\,Y_{\mathrm{BAt}}[R_{\mathrm{PA}}(t, T) - R_{\mathrm{BA}}(t, T)] \\ \text{interaction} \;=\; I_{\mathrm{A}} &= \Sigma_{\mathrm{t=0}}{}^{\mathrm{T}}\,(Y_{\mathrm{PAt}} - Y_{\mathrm{BAt}})[R_{\mathrm{PA}}(t, T) - R_{\mathrm{BA}}(t, T)] \end{aligned} \qquad (2)$$

Active asset allocation decisions will mean that the cash inflows and outflows for a given asset class will differ between portfolio and benchmark, ie $Y_{\mathrm{PAt}} \neq Y_{\mathrm{BAt}}$. In the case of a switch between asset classes, with no cash flow by investors, Y_{PAt} will be non-zero for the relevant asset classes, whereas Y_{BAt} will be zero.

We can now explain why attribution analysis is easier using values than it is using TWRs, whether or not there are intervening cash flows. The value added can be analyzed exactly both at the level of the fund and at the level of a particular asset class, without making adjustments to any values or returns. The effect of each attribute is measured by a final value added or lost in relation to the benchmark, as shown in equations (2). The values for each attribute add up to the value added or lost for the whole portfolio: summing across the N asset classes, $\Sigma_{\mathrm{A=1}}{}^{\mathrm{N}}\,Y_{\mathrm{PAt}} = \Sigma_{\mathrm{A=1}}{}^{\mathrm{N}}\,Y_{\mathrm{BAt}} = Y_{\mathrm{t}}$ for each date t, and the formulas in equations (2) reduce to

$$\begin{aligned} \Sigma_{\mathrm{A=1}}{}^{\mathrm{N}}\,\Sigma_{\mathrm{t=0}}{}^{\mathrm{T}}\,(AA_{\mathrm{At}} + SS_{\mathrm{At}} + I_{\mathrm{At}}) &= \Sigma_{\mathrm{t=0}}{}^{\mathrm{T}}\,Y_{\mathrm{t}}[R_{\mathrm{P}}(t, T) - R_{\mathrm{B}}(t, T)] \\ &= V_{\mathrm{PT}} - V_{\mathrm{BT}}. \end{aligned}$$

In comparison, $R_{\mathrm{P}} - R_{\mathrm{B}}$ or $(1 + R_{\mathrm{P}})/(1 + R_{\mathrm{B}})$ can not be analyzed exactly without first adjusting the returns for each period, as discussed in the Introduction and in note 2.

There is one problem which the value-based method does not avoid: the attribution results for the T periods combined can not be arrived at by adding the results for each period analyzed in isolation. A correct attribution analysis can be carried out for period 1, because both a portfolio and its benchmark start with the same value. But a correct attribution analysis for period 2 would involve re-setting the benchmark at the start of period 2 so that it had the same value as the portfolio, and so on. The final values of the benchmark and of its constituent asset classes would also need to be re-set, in which case they would no longer match the true final values for the benchmark. This alternative procedure would provide correct attribution values for each period, but the sum of these values would not result in a correct analysis of the value added or lost across all the periods together (though it would normally be a very close approximation). The same problem arises using TWRs: the attribution analysis of the difference in TWRs over T periods, $R_P - R_B$, can not be reconciled with the analysis of $R_{Pt} - R_{Bt}$ for any given period t in T, unless an adjustment is first made to the returns for each period.

Data requirements

Ideally the market values of the asset classes of the managed portfolio and of its benchmark should be measured each time there is a cash flow by an investor or a change in asset allocation by the manager. The closer the time of the valuation to the time of the cash flow or re-allocation, the more accurate will be the results. It is not necessary to know the values of asset classes in between the times of cash flows and re-allocations. Daily valuations, for example, are not needed unless the fund in question does in fact have cash flows or asset re-allocations on most days. The same comments apply to analysis using TWRs. So the data requirements are the same for measurement by values as for measurement by TWRs.

Example 1A: no intervening cash flow; one investor

We now illustrate the value-based method by means of an example. Table 1 shows the composition and performance of a managed portfolio and of its benchmark over two periods. We assume temporarily that there is no new money invested at the end of period 1 (= date 1). There are three asset classes available; equity, bonds and cash. The portfolio has an initial value of $1,000 invested 55% in equity with 45% in bonds. The benchmark is different; the same $1,000 is invested 50% in equity, 40% in bonds and 10% in cash. The returns for the portfolio are 4.48% in period 1 and –2.80% in period 2, giving a TWR of 1.55%. The TWR for the benchmark is 2.12%, so the portfolio has underperformed by –0.56% [= (0.0155 – 0.0212)/1.0212]. In terms of values, the portfolio has a final value of $1,015.46, compared with $1,021.20 for the benchmark. The shortfall of –$5.74, expressed as a percentage of the benchmark value is –0.56%, exactly the same as the underperformance derived from the two TWRs.

Table 1 around here

Example 1B: intervening cash flow; one investor

We now assume that a further \$200 is invested at date 1. The portfolio manager decides to hold this \$200 in cash for the second period, but the notional \$200 invested in the benchmark is allocated amongst the asset classes in line with the weighting of each class in the benchmark at date 1. Table 2 shows the benchmark for the new money, and the performance of the new money compared with its benchmark. The composition of the benchmark in period 2 is the same for the new money as for the existing investment, and has the same return over period 2 as above in Table 1. The portfolio's return on the new money is quite different, since all \$200 was retained in cash and earned 1.25% rather than the –1.95% which the benchmark earned. The 'portfolio' consisting of this new cash alone outperformed the benchmark by +3.26% in period 2, whether calculated from the returns in period 2 or from the portfolio's value added of \$6.40 divided by its benchmark value of \$196.10.

Tables 2 and 3 around here

The next step is to add together the values relating to the initial investment and to the new cash. This sum is shown in Table 3. The new cash changes the value of the benchmark but not its composition, so the benchmark including the new cash has the same return in period 2 as in Tables 1 and 2, and the same two-period return of 2.12%. But the return for the portfolio including the new cash is now –2.15% in period 2, rather than –2.80%. This is the result of combining the period 2 return of –2.80% on the initial investment of \$1,000 with the period 2 return of 1.25% on the new investment of \$200 at date 1. The TWR for the total portfolio is now 2.23%, rather than 1.55%, and the relative performance is (0.0223 – 0.0212)/1.0212 = +0.10%. In values, the portfolio is worth \$1,217.96 compared with \$1,217.30 for the benchmark, a difference of +\$0.66. This difference expressed as a percentage of the benchmark value gives the relative performance of +0.05%. Now that we have an intervening cash flow, the performance measure in relation to the benchmark derived from TWRs does not match the measure derived from values. This means that attribution analysis which seeks to account for the generation of excess value of +\$0.66 can not be done exactly using TWRs. The reason for the mismatch is that TWRs give equal weighting to the returns in each period, whereas in reality the return in period 2 has more impact on the final value, because there was extra cash invested during period 2.

It is true that, in this example, both the value-based and TWR-based methods result in a similar inference, which is that the managed portfolio did slightly better than its benchmark during the two periods. But the discrepancy between the relative performance measures would widen were the intervening cash flows larger. For example, if $10,000 were invested at date 1 instead of $200, relative performance would be +3.19% using TWRs and +2.90% using values. More dramatically, the inferences from the methods can diverge, because they can differ in sign. That is, the relative performance using TWRs can be positive, yet value has been lost in relation to the benchmark, and vice versa.

Example 1C: intervening cash flow; two investors

To see how the value-based method enables investor-specific attribution to be carried out, and to see how a difference in sign can arise compared with the TWR measure, suppose now that there were two investors in the portfolio, as shown in Table 4. Investor X placed $900 with the manager at date 0, with no further payments, while Investor Y placed $100 at date 0 and was responsible for the entire cash inflow of $200 at date 1. The composition of the whole portfolio, and the returns on each asset class, are the same as before.

Table 4 around here

The situation in period 1 is that X has 90% and Y has 10% of the managed portfolio, so both earn the return of the portfolio in period 1, 4.48%, and the benchmark return for both is the benchmark return for the portfolio, 4.15%. The situation in period 2 needs a little more explanation. After Y's investment at date 1, the composition of *both* portfolios changes. After period 1 but before Y's investment, the managed portfolio consists of 56% equity, 44% bonds and 0% cash; X owns 90% and Y owns 10%. After Y's investment, the portfolio consists of 47% equity, 37% bond and 16% cash; X owns 76% and Y owns 24%. Thus, even though X does nothing, the composition of his portfolio changes, as does Y's, because of the manager's decision to keep the new $200 in cash. For example, X now has $151.08 in cash. The composition of the benchmark portfolio does not change as a result of Y's investment, because the new cash is allocated in line with the existing proportions, which are 51% equity, 39% bonds and 10% cash at date 1.

X's initial $900 in the managed portfolio grew to $920.04 by date 2. If the $900 had been invested in the benchmark, then the final value would have been $919.08. Thus the manager added value for X in relation to the benchmark of $0.96, giving a relative performance of +0.10%. The outcome for Y is different. The final value for Y is $297.92, compared with \$298.22 had Y's money at dates 0 and 1 been invested in the benchmark, so the manager lost \$0.30, giving a relative performance of –0.10%. In this case Y also suffered an absolute capital loss: the \$100 invested at date 0 plus the \$200 at date 1 is only worth \$297.92 at date 2.

The results using TWRs are those for the whole portfolio in Table 3, which means that X and Y both have a TWR of +2.23%, and that both have a relative performance of +0.10%. X's portfolio had no intervening cash flow, so the absolute value did indeed increase by 2.23%, and the value-based and TWR-based methods produce the same relative performance figure of +0.10%. However, the results from the TWR-based method for Y's portfolio are misleading. The relative performance for Y from TWRs is +0.10%, but in fact Y's portfolio lost value in relation to the benchmark. The compound return for Y is 2.23%, but in fact Y suffered a capital loss. The explanation is that the managed portfolio did badly in period 2 both absolutely and relatively, and TWRs do not capture the fact that Y invested more money in it at the start of period 2.

Attribution analysis in Example 1C

The aim of attribution analysis using values is to explain the difference between the value of an investor's portfolio and that of its benchmark at the final date. This difference is the manager's contribution; it is additional to the impact on the benchmark of the size and timing of the cash flows. For example, had Y's end-date benchmark value been calculated on the assumption that the \$200 was invested at date 0, rather than date 1, then the benchmark value would have been \$306.36 rather than \$298.92. The difference of –\$7.44 in the benchmark represents a loss attributable to the later actual payment of \$200 at date 1. Since the portfolio's performance is judged against the lower benchmark value of \$298.92, the loss is accountable to the investor rather than the manager.

Tables 5 around here

Table 5 shows an attribution analysis for the total portfolio and for each investor. Readers can see from this table how attribution using values works out numerically, and can further satisfy themselves that it delivers the precision and transparency which we claim for it. The underlying formulas are given by equations (2).

The attribution calculation in the first panel of Table 5 is for the total portfolio. The value-based method allows for customized reports to be produced, one for each investor, as shown in the second and third panels. The values in these panels sum to give the values for the total portfolio. The analysis explains the manager's contribution of $0.96 for X and –$0.30 for Y, relative to the benchmark. It is precise, accurate to the cent; there are no 'buckets' of unexplained contributions to performance for either investor.

The example also illustrates how an absence of holding in an asset class can be handled. Table 5 shows the contribution from the absence of cash in the portfolio at the start, in relation to the benchmark which has 10% in cash.[6]

It should be clear that exactly the same value-based analysis can be done at the level of individual assets, instead of asset classes, although naturally the data requirements are greater. It will also be appreciated that it is straightforward to accommodate expenses, tax payments and short positions. They involve cash outflows from a portfolio which can be identified and dated. We do not pursue these aspects of performance appraisal here, but turn instead to the case of a multicurrency portfolio.

Currency analysis

The currency dimension adds complexity to the task of performance measurement and analysis. To demonstrate the flexibility of the value-based method, we now develop a second example which involves holdings in more than one currency.

Example 2: two-currency portfolio with cross-currency switch

Table 6 shows the composition and performance of an international portfolio and of its benchmark over two periods. For simplicity there are only two asset classes, holdings in the USA and holdings in the UK, and there are no intervening cash flows in this example. As in Example 1, the performance in relation to the benchmark is the difference between the final value of the portfolio and of its benchmark, which in this case is –$2.07, or –0.18% of the final value of the notional holding in the benchmark. Since there is no intervening cash flow, the relative performance is exactly the same using TWRs.

Tables 6 and 7 around here

An attribution analysis is shown in Table 7 for the two periods. Our analysis breaks down the value added into three primary components and four interaction terms. The three primary components are market selection and stock selection, both ignoring currency, and currency selection, which captures the exchange rate gain or loss from market selection. The market selection and stock selection attributes for a cash flow for market M, together with the first interaction term, are arrived at by ignoring any change in the exchange rate:

$$\begin{aligned}\text{market selection (MS)} &= (Y_{\text{PMt}} - Y_{\text{BMt}})[R_{\text{BM}}(t, T) - R_{\text{B}}(t, T)]\\ \text{stock selection (SS)} &= Y_{\text{BMt}}[R_{\text{PM}}(t, T) - R_{\text{BM}}(t, T)]\\ \text{MS.SS} &= (Y_{\text{PMt}} - Y_{\text{BMt}})[R_{\text{PM}}(t, T) - R_{\text{BM}}(t, T)],\end{aligned}$$

where M stands for a market and the returns are calculated as though exchange rates were fixed. These equations are the same as equations (2), except that a market M has replaced an asset class A. The currency selection attribute is given by

$$\text{currency selection (CS)} = (Y_{\text{PM}} - Y_{\text{BM}})E(t, T),$$

where $E(t, T)$ is the percentage change in the exchange rate between the currency of the relevant market and the portfolio's master currency between dates t and T. The three further interaction terms are

$$\begin{aligned}\text{MS.CS} &= (Y_{\text{PMt}} - Y_{\text{BMt}})R_{\text{BM}}(t, T)E(t, T)\\ \text{SS.CS} &= Y_{\text{BMt}}[R_{\text{PM}}(t, T) - R_{\text{BM}}(t, T)]E(t, T)\\ \text{MS.SS.CS} &= (Y_{\text{PMt}} - Y_{\text{BMt}})[R_{\text{PM}}(t, T) - R_{\text{BM}}(t, T)]E(t, T).\end{aligned}$$

These terms capture the interaction between the change in the exchange rate and the local-currency value gained or lost through, respectively, market selection, stock selection and the cross-product of these. The seven attribution terms sum to give the manager's contribution relating to a given cash flow Y_{PMt}. The contribution is the change in $Y_{\text{PMt}} - Y_{\text{BMt}}$ between dates t and T, less the change which would have arisen had the return on M in P been the same as the overall return on the benchmark, ignoring currency. In symbols,

$$\begin{aligned}V_{\text{added}}(Y_{\text{PMt}}) &= \text{MS} + \text{SS} + \text{CS} + \text{MS.SS} + \text{MS.CS} + \text{SS.CS} + \text{MS.SS.CS}\\ &= \{Y_{\text{PMt}}[1 + R_{\text{PM}}(t, T)] - Y_{\text{BMt}}[1 + R_{\text{BM}}(t, T)]\}[1 + E(t, T)]\\ &\quad - (Y_{\text{PMt}} - Y_{\text{BMt}}) - R_{\text{B}}(t, T)(Y_{\text{PMt}} - Y_{\text{BMt}}).\end{aligned}$$

In the case of a market denominated in the portfolio's master currency, $E(t, T) = 0$, and the analysis reduces to the analysis for an asset class, as in the single-currency case.[7]

The clear and unequivocal calculation of the interaction terms is a particular benefit of the value approach. There is no reason to factor these numbers 'back into' the three principal numbers of markets, currency and stock selection; the values of the interaction terms are kept separate from the principal components. Managers usually decide on markets, currency and stocks and are not thinking about interaction terms. Correct attribution of the principal components of performance should not be muddied by the second-order interaction terms which, if factored back in, can seriously change the principal components, even turning a positive contribution negative or vice versa.

A problem can arise for TWRs when there is a negative market value, for example from going short, using derivatives or from selling assets. In our example there is a switch from the UK to the US at date 1, involving a negative market value for UK assets. Using the value-based method, it is possible to quantify and analyze the gain, or in this case the loss, arising from the switch, something which is not possible using TWRs. Table 8 shows the attribution analysis in value terms for the $5.90 loss from the switch.

Table 8 around here

A further problem in currency attribution is created by 'currency tainting'. For example, suppose that the benchmark of a Far Eastern equity portfolio, reporting in US$, is the relevant component of the FTSE All-World Index. Some of the Far Eastern portfolio is held in assets denominated in HK$, which is tied to the US$. The currency-related performance of the benchmark is determined primarily by the Yen/$ exchange rate. If the Yen falls against the US$, the HK$ holding will show a positive currency contribution, when in fact there is no currency benefit from being in the HK$. This problem is avoided using values, since the value of the portfolio's holding in a currency is assessed against the value of the benchmark's holding, both expressed in the master currency. Thus, there would be a currency contribution of zero for the HK$ holding, which is correct.

Conclusion

The method presented here proposes a value-based measure of the contribution of the fund manager to an investor's holding over a given length of time. The measure is the final value added or lost by the manager in relation to a benchmark, and is calculated net of the effects of cash inflows and outflows made by the investor. We have argued that it is preferable to measure the manager's contribution using values and to measure the manager's skill and make cross-fund comparisons using TWRs. Since the manager's contribution and skill are both, we are sure, of interest to investors, value-based and returns-based measurement should be seen as complements rather than substitutes. We have shown that multiperiod attribution analysis is precise and transparent using values, and is in fact easier than the corresponding analysis using TWRs. We hope that the paper will encourage performance measurers to explore the use of value-based analysis further.

Notes

1. Contributions include Cariño (1999), Frongello (2002) and Menchero (2000a).

2. For example, the geometric measure for a single period t can be analyzed as follows (Burnie et al, 1998; Bacon, 2002):

$$1 + G_t = (1 + R_{Pt})/(1 + R_{Bt}) = (1 + {}_{AA}R_t)(1 + {}_{SS}R_t)$$

where ${}_{AA}R_t$ and ${}_{SS}R_t$ are the incremental returns due to asset allocation and stock selection, respectively:

$$1 + {}_{AA}R_t = \Sigma_{A=1}^{n} (w_{PAt} - w_{BAt})[(R_{BAt} - R_{Bt})/(1 + R_{Bt})]$$

$$1 + {}_{SS}R_t = \Sigma_{A=1}^{n} w_{PAt}(R_{PAt} - R_{BAt})/(1 + R_{Bt}*),$$

where A is an asset class, w is a weight, and $R_{Bt}* = \Sigma_{A=1}^{n} w_{PAt}R_{BAt}$. (The interaction effect is contained in stock selection under this definition.) It is then the case that

$$1 + G = \Pi_{t=1}^{T} (1 + {}_{AA}R_t)(1 + {}_{SS}R_t).$$

But

$$1 + {}_{AA}R_A \neq \Pi_{t=1}^{T} (1 + {}_{AA}R_{At})$$

and

$$1 + {}_{SS}R_A \neq \Pi_{t=1}^{T} (1 + {}_{SS}R_{At}),$$

where ${}_{AA}R_A$ and ${}_{SS}R_A$ are the would-be incremental compound returns due to asset allocation and stock selection, respectively, for asset class A. In other words, the multiperiod incremental returns due to asset allocation or stock selection for a particular asset class can not be found by compounding the single-period incremental returns due to asset allocation or stock selection, respectively, for this asset class. Menchero (2000b) and Mirabelli (2000) present alternative solutions to this problem.

3. The concept of a money-weighted return applies to a period during which there are one or more cash inflows or outflows (see, for example, Darling and MacDougall, 2002, or Spaulding, 2001). Our method requires that the benchmark be valued at the time of each cash inflow and outflow, so in our analysis there are no cash flows within any of the T periods. This means we never use money-weighted returns.

4. The difference between the internal rates of return (IRRs) for a managed fund and its benchmark is, like the difference in the final values, an exact measure of the contribution of the manager. Use of IRRs has recently been discussed and recommended by Illmer and Marty (2003). But there are severe problems with an IRR-based performance measure. IRR can not be calculated unambiguously for a fund or asset class with one or more cash outflow and then inflow after the initial investment and before the final date. This is the change-of-sign

problem, and it rules out use of IRR for many funds, for example hedge funds with short positions. In addition, there is no exact method of attribution using IRRs that we know of.

5. A fuller account would include rules regarding treatment of cash receipts from individual assets, of cash flows arising from use of derivatives, and of expenses and taxes.

6. A void in a period other than the first is harder to deal with. For the purpose of calculating the multiperiod value added from stock selection and interaction for the asset class, it is necessary to insert a notional return for the period(s) with a nil holding. We argue that this notional return should be such that the accumulated value added or lost from stock selection on an earlier cash flow is preserved into the future. The formula is

$$1 + R_\tau(Y_{BAt}) = \{R_{PA}(t, \tau - 1) - R_{BA}(t, \tau - 1) + [1 + R_{BA}(t, \tau - 1)](1 + R_{BA\tau})\}/[1 + R_{PA}(t, \tau - 1)],$$

where the portfolio is void in asset class A in period τ, $R_{PA\tau}(Y_{BAt})$ is the notional return on A for period τ which should be entered for a cash flow at an earlier date t, $R_{PA}(t, \tau - 1)$ is the compound return on the asset class in the portfolio between dates t and $\tau - 1$, and $R_{PA}(t, \tau - 1)$ is the same for the asset class in the benchmark. For example, consider the following returns for an asset class

	1	2	3
Portfolio	6%	6%	void
Benchmark	4%	4%	4%

The gain from stock selection by date 2 relating to cash of Y_{BA0} is $Y_{BA0}[(1.06)^2 - (1.04)^2] = Y_{BA0}(4.2\%)$. The notional return on A in the portfolio in period 3 to carry this gain to date 3 is

$$1 + R_3(Y_{BA0}) = [0.1236 - 0.0816 + (1.0816)(1.04)]/(1.1236)$$

$$R_3(Y_{BA0}) = 3.85\%,$$

ie $4.2\% = (1.06)^2(1.0385) - (1.04)^3$. Similarly, the notional return on A in the portfolio in period 3 for the purpose of calculating the gain at date 3 from stock selection relating to cash at date 1 is

$$1 + R_3(Y_{BA1}) = [0.06 - 0.04 + (1.04)(1.04)]/(1.06)$$

$$R_3(Y_{BA1}) = 3.92\%.$$

7. Other types of analysis are possible. For example, Ankrim and Hensel (1994) and Singer and Karnosky (1995) present analyses which isolate the effect of the interest rate parity relation between exchange and interest rates. Ankrim and Hensel do this by splitting the currency selection component into a ‘forward premium effect’ and a term for the difference between the actual change in the exchange rate and the relevant forward premium. Singer and

Karnosky deduct the local-currency eurodeposit rate from the market return in the ‘market selection’ and ‘stock selection’ components. ‘Currency selection’ is then defined as the sum of the local-currency euro-rate and the change in the exchange rate. Both papers also allow for currency hedging. We have not isolated the effect of the interest rate parity relation, nor included a hedged position, in order to keep the example as simple as possible. There is no problem in conducting either an Ankrim-Hansel or a Singer-Karnosky analysis in value terms.

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Table 1. Example 1A: A managed portfolio and its benchmark over two periods

	Date 0 Compos-ition	*Period 1* Return	*Date 1* Compos-ition	*Date 1* Allocation of new cash	*Period 2* Return	*Date 2* Compos-ition	Compound return	Value added	Difference in value added
Managed portfolio									
Equity	$550.00	6.50%	$585.75	$0.00	–5.00%	$556.46			
Bonds	$450.00	2.00%	$459.00	$0.00	0.00%	$459.00			
Cash	$0.00	n/a	$0.00	$0.00	1.25%	$0.00			
Total	$1,000.00	4.48%	$1,044.75	$0.00	–2.80%	$1,015.46	1.55%	$15.46	
Benchmark portfolio									
Equity	$500.00	6.00%	$530.00	$0.00	–4.60%	$505.62			
Bonds	$400.00	2.50%	$410.00	$0.00	0.50%	$412.05			
Cash	$100.00	1.50%	$101.50	$0.00	2.00%	$103.53			
Total	$1,000.00	4.15%	$1,041.50	$0.00	–1.95%	$1,021.20	2.12%	$21.20	–$5.74
Relative performance[1]							–0.56%		–0.56%

Note
1. Relative performance is given by $[R_P(0, 2) - R_B(0, 2)]/[1 + R_B(0, 2)] - 1$ for returns, and by $(V_{P2} - V_{B2})/V_{B2}$ for values, where V_2 is the value of portfolio P or benchmark B at date 2.

Table 2. Example 1B: New cash for period 2

	Date 1 Benchmark proportions[1]	*Date 1* Allocation of new cash	*Period 2* Return	*Date 2* Compos-ition	Return	Value added	Difference in value added
Managed portfolio							
Equity		$0.00	–5.00%	$0.00			
Bonds		$0.00	0.00%	$0.00			
Cash		$200.00	1.25%	$202.50			
Total		$200.00	1.25%	$202.50	1.25%	$2.50	
Benchmark portfolio							
Equity	50.89%	$101.78	–4.60%	$97.09			
Bonds	39.37%	$78.73	0.50%	$79.13			
Cash	9.75%	$19.49	2.00%	$19.88			
Total	100.0%	$200.00	–1.95%	$196.10	–1.95%	–$3.90	$6.40
Relative performance					3.26%		3.26%

Note
1. The benchmark proportions are the values of each asset class in the benchmark at date 1 divided by the total value of the benchmark at date 1, before the new cash arrives (from Table 1).

Table 3. Example 1B: The managed portfolio with the new cash

	Date 0	*Period 1*	*Date 1*	*Date 1*	*Period 2*	*Date 2*			
	Compos-ition	Return	Compos-ition	Allocation of new cash	Return	Compos-ition	Compound return	Value added	Difference in value added
Managed portfolio									
Equity	$550.00	6.50%	$585.75	$0.00	–5.00%	$556.46			
Bonds	$450.00	2.00%	$459.00	$0.00	0.00%	$459.00			
Cash	$0.00	n/a	$0.00	$200.00	1.25%	$202.50			
Total	$1,000.00	4.48%	$1,044.75	$200.00	–2.15%	$1,217.96	2.23%	$17.96	
Benchmark portfolio									
Equity	$500.00	6.00%	$530.00	$101.78	–4.60%	$602.71			
Bonds	$400.00	2.50%	$410.00	$78.73	0.50%	$491.18			
Cash	$100.00	1.50%	$101.50	$19.49	2.00%	$123.41			
Total	$1,000.00	4.15%	$1,041.50	$200.00	–1.95%	$1,217.30	2.12%	$17.30	$0.66
Relative performance							0.10%		0.05%

Table 4. Example 1C: The managed portfolio with the new cash and two investors

	Date 0 Investors' holdings	*Period 1* Return	*Date 1* Holdings before new cash	*Date 1* Holdings after new cash	*Period 2* Return	*Date 2* Investors' holdings	Compound return	Value added	Difference in value added	Relative perform-ance
Investor X's portfolio										
Equity	$495.00	6.50%	$527.18	$442.47	–5.00%	$420.35				
Bonds	$405.00	2.00%	$413.10	$346.73	0.00%	$346.73				
Cash	$0.00	n/a	$0.00	$151.08	1.25%	$152.97				
Total for X	$900.00	4.48%	$940.28	$940.28	–2.15%	$920.04	2.23%	$20.04		
Investor Y's portfolio										
Equity	$55.00	6.50%	$55.58	$143.28	–5.00%	$136.12				
Bonds	$45.00	2.00%	$45.90	$112.27	0.00%	$112.27				
Cash	$0.00	n/a	$0.00	$48.92	1.25%	$49.53				
Total for Y	$100.00	4.48%	$104.48	$304.48	–2.15%	$297.92	2.23%	-$2.08		
Total	$1,000.00	4.48%	$1,044.75	$1,244.75	–2.15%	$1,217.96	2.23%	$17.96		
Investor X's benchmark										
Equity	$450.00	6.00%	$477.00	$477.00	–4.60%	$455.06				
Bonds	$360.00	2.50%	$369.00	$369.00	0.50%	$370.85				
Cash	$90.00	1.50%	$91.35	$91.35	2.00%	$93.18				
Total for X	$900.00	4.15%	$937.35	$937.35	–1.95%	$919.08	2.12%	$19.08	$0.96	0.10%
Investor Y's benchmark										
Equity	$50.00	6.00%	$53.00	$154.78	-4.60%	$147.66				
Bonds	$40.00	2.50%	$41.00	$119.73	0.50%	$120.33				
Cash	$10.00	1.50%	$10.15	$29.64	2.00%	$30.23				
Total for Y	$100.00	4.15%	$104.15	$304.15	–1.95%	$298.22	2.12%	-$1.78	-$0.30	-0.10%
Total	$1,000.00	4.15%	$1,041.50	$1,241.50	–1.95%	$1,217.30	2.12%	$17.30	$0.66	0.05%

Table 5. Example 1C: Performance attribution for total portfolio and for portfolios of each investor

	Asset allocation	Stock selection	Asset allocation x selection	Total
Total portfolio				
Equity	$2.20	–$0.15	$0.43	$2.48
	($550.00 – $500.00)(1.12% – 2.12%) + ($0.00 – $101.78)(–4.60% + 1.95%)	*$500.00(1.17% – 1.12%) + $101.78(–5.00% + 4.60%)*	*($550.00 – $500.00)(1.17% – 1.12%) + ($0.00 – $101.78)(–5.00% + 4.60%)*	
Bonds	–$1.48	–$4.44	–$0.11	–$6.04
	($450.00 – $400.00)(3.01% – 2.12%) + ($0.00 – $78.73)(0.50% + 1.95%)	*$400.00(2.00% – 3.01%) + $78.73(0.00% – 0.50%)*	*($450.00 – $400.00)(2.00% – 3.01%) + ($0.00 – $78.73)(0.00% – 0.50%)*	
Cash	$5.72	–$0.91	–$0.59	$4.22
	($0.00 – $100.00)(3.53% – 2.12%) + ($200.00 – $19.49)(2.00% + 1.95%)	*($101.50 + $19.49)(1.25% – 2.00%)*	*($200.00 – ($101.50 + $19.49)) (1.25% – 2.00%)*	
Total	$6.44	–$5.50	–$0.27	$0.66
X's portfolio				
Equity	$1.80	$0.23	$0.36	$2.39
	0.9($550.00 – $500.00)(1.12% – 2.12%) + (–$84.70 – $0.00)(–4.60% + 1.95%)	*0.9($500.00)(1.17% – 1.12%) + $0.00*	*0.9($550.00 – $500.00)(1.17% – 1.12%) + (–$84.70 – $0.00)(–5.00% + 4.60%)*	
Bonds	–$1.22	–$3.64	–$0.12	–$4.99
	0.9($450.00 – $400.00)(3.01% – 2.12%) + (–$66.37 – $0.00)(0.50% + 1.95%)	*0.9($400.00)(2.00% – 3.01%) + $0.00*	*0.9($450.00 – $400.00)(2.00% – 3.01%) + (–$66.37 – $0.00)(0.00% – 0.50%)*	
Cash	$4.70	–$0.69	–$0.45	$3.56
	0.9($0.00 – $100.00)(3.53% – 2.12%) + ($151.08 – $0.00)(2.00% + 1.95%)	*($91.35 + $0.00)(1.25% – 2.00%)*	*($151.08 – $91.35)(1.25% – 2.00%)*	
Total	$5.27	–$4.10	–$0.21	$0.96
Y's portfolio				
Equity	$0.40	–$0.38	$0.07	$0.09
	0.1($550.00 – $500.00)(1.12% – 2.12%) + ($84.70 – $101.78)(–4.60% + 1.95%)	*0.1($500.00)(1.17% – 1.12%) + $101.78(–5.00% + 4.60%)*	*0.1($550.00 – $500.00)(1.17% – 1.12%) + ($84.70 – $101.78)(–5.00% + 4.60%)*	
Bonds	–$0.26	–$0.80	$0.01	–$1.05
	0.1($450.00 – $400.00)(3.01% – 2.12%) + ($66.37 – $78.73)(0.50% + 1.95%)	*0.1($400.00)(2.00% – 3.01%) + $78.73(0.00% – 0.50%)*	*0.1($450.00 – $400.00)(2.0% – 3.01%) + ($66.37 – $78.73)(0.00% – 0.50%)*	
Cash	$1.02	–$0.22	–$0.14	$0.65
	0.1($0.00 – $100.00)(3.53% – 2.12%) + ($48.92 – $19.49)(2.00% + 1.95%)	*($10.15 + $19.49)(1.25% – 2.00%)*	*($48.92 – $29.64)(1.25% – 2.00%)*	
Total	$1.17	–$1.40	–$0.06	–$0.30

Note: See text for formulas.

Table 6. Example 2: A multicurrency portfolio and its benchmark over two periods

	Date 0	*Period 1*	*Date 1*	*Date 1*	*Period 2*	*Date 2*			
Exchange rate $/£	1.60		1.50			1.57			
	Compos-ition	Return (in local currency)	Compos-ition	Re-allocation of portfolio	Return (in local currency)	Compos-ition	Compound return	Value added	Difference in value added
Managed portfolio									
US assets	$800.00	10.00%	$880.00	$100.00	4.00%	$1,019.20			
UK assets	$200.00	8.00%	$202.50	–$100.00	5.00%	$112.65			
Total	$1,000.00	8.25%	$1,082.50	$0.00	4.56%	$1,131.85	13.18%	$131.85	
Benchmark portfolio									
US assets	$600.00	9.00%	$654.00	$0.00	5.00%	$686.70			
UK assets	$400.00	8.00%	$405.00	$0.00	5.50%	$447.21			
Total	$1,000.00	5.90%	$1,059.00	$0.00	7.07%	$1,133.91	13.39%	$133.91	–$2.07
Relative performance							–0.18%		–0.18%

Table 7. Example 2: Performance attribution

				Cross-product terms				
	Market selection	Stock selection	Currency selection	Market selection x stock selection	Market selection x currency change	Stock selection x currency change	Market selection x stock sel'n x currency change	Difference in value added
US assets	$0.21	–$0.30	n/a	–$1.10	n/a	n/a	n/a	–$1.19
UK assets	$0.31	–$2.16	–$0.92	$1.58	$0.27	$0.04	$0.00	–$0.88
Total	$0.52	–$2.46	–$0.92	$0.48	$0.27	$0.04	$0.00	–$2.07

Note: See text for formulas.

Table 8. Example 2: Performance attribution for cross-currency switch

	Date 1	*Period 2*	*Date 2*								
Exchange rate $/£	1.50		1.57								
	Value of switch	Return	Values	Market selection	Stock selection	Currency selection	Market selection x stock selection	Market selection x currency change	Stock selection x currency change	Market selection x stock sel'n x c'y change	Difference in value added
US assets	$100.00	4.00%	$104.00	–$0.20	$0.00	n/a	–$1.00	n/a	n/a	n/a	–$1.20
UK assets	–$100.00	5.00%	–$109.90	–$0.30	$0.00	–$4.67	$0.50	–$0.26	$0.00	$0.02	–$4.70
Total	$0.00		–$5.90	–$0.50	$0.00	–$4.67	–$0.50	–$0.26	$0.00	$0.02	–$5.90

Note: See text for formulas.